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On the Solar Dynamo Theory

KEITH LEON McDONALD

BOULDER, COLO.
JULY 1970

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ON THE SOLAR DYNAMO THEORY

Keith Leon McDonald

A spherical harmonic expansion of the mean velocity field may be estimated in terms of poloidal, toroidal, and scaloidal vector sets. The angular velocity at the photospheric level remains fairly independent of the radial coordinate throughout the chromosphere and well up into the corona, and the azimuthal velocity is analyzed into zonal toroidal vectors of odd degree. The mean meridional motions below the photospheric level resemble a poloidal quadrupole mode, $\underset{\sim}{P}_{2c}^{0}$, with motion towards the equator, where a general subsidence occurs with augmented modulus. The two principal zonal toroidal magnetic fields of each hemisphere are circulated to greater depths by about 12-15 earth radii, allowing for an overlap of one year between semicycles, and ascend near latitudes ±40°. The analysis of meridional velocities into poloidal and scaloidal components is difficult because it requires a knowledge of the radial dependency of both radial and polar components. A subsidiary circulation is postulated in the corona to account for the poleward drift of coronal filaments (5 m sec^{-1}), with a near photospheric quasi-stagnation point. Filaments drift poleward from 30-40° latitudes where they are produced during the beginning of the sunspot semi-cycle, to about 70-80° latitudes, and from 10-15° at the end of the semi-cycle to about 45-55°, therefore enduring some 30-40 solar rotations. Only the regenerative feedback mechanism is the same as in planetary dynamos, the amplifying mechanism now requiring a compressible medium; the poloidal field lines are "stretched" in the azimuthal direction by the θ-dependent turbulent differential rotation into toroidal field lines, while at the deeper levels the lines are crowded closer by compression and the field is amplified to larger values, in a region of greater electrical conductivity. An assemblage of magnetic lines approaching the equator is circulated to greater depths and after inversion and compression it is re-amplified as a similar configuration. Phase-synchronism of the circulating toroidal magnetic modes is maintained sensibly by Lorentz forces thru self-interaction; the leading magnetic assemblage tends to move across the equatorial plane thereby delaying its movement by increasing the circuital path length. The magnetic fields that are diffused into the meridional circulations at greater depths are transported to higher latitudes where they emerge to engender the polar fields.

1. INTRODUCTION

The present work contains a rough description of the basic mechanisms involved in the solar dynamo, together with a description of the large-scale or secular velocity fields and the principal magnetic fields. The large body of observational data that has accumulated during the past two or three decades on both the velocity and magnetic fields has prompted the construction of our present model, and supplies tentative confirmation of its correctness, although uniqueness is not necessarily claimed. Many observed phenomena still remain obscure but there seem to be no serious contradictions.

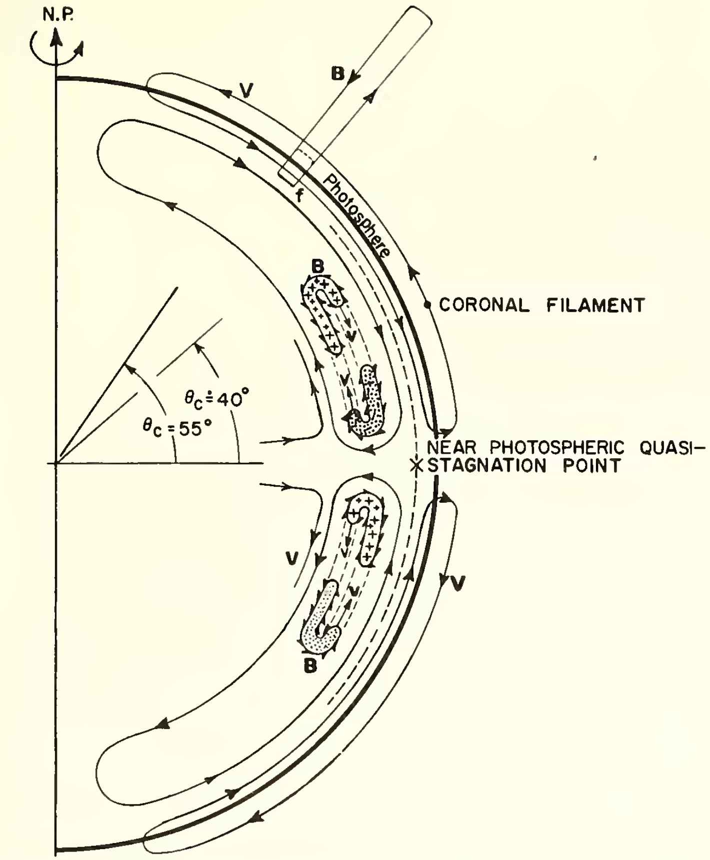
In spite of the strong toroidal magnetic fields found in the underlying layers below the photosphere (up to 4,000 gauss, according to the field intensities observed in sunspots), the large scale fluid motions are generally controlled by other kinds of forces—Coriolis, ponderomotive, and geometrical—and the magnetic field has served the role of a passive indicator of the fluid motion rather than

that of prime mover. The source of all fluid motions is thermal convection. Acted upon by Coriolis forces, the thermal convection engenders both a mean velocity field in the azimuthal direction, known as the <u>turbulent differential rotation</u>, and a meridional circulation that occupies about 15% of the outermost radius of the sun below the photosphere. Both secular motions necessarily coexist. The influence of the meridional circulation on the azimuthal velocity distribution is manifested by the magnetic field; large recurrent sunspots are dragged by the more rapidly flowing ambient fluid, and prominences move more rapidly than the underlying photospheric matter as a result of their poleward proper motions, etc.

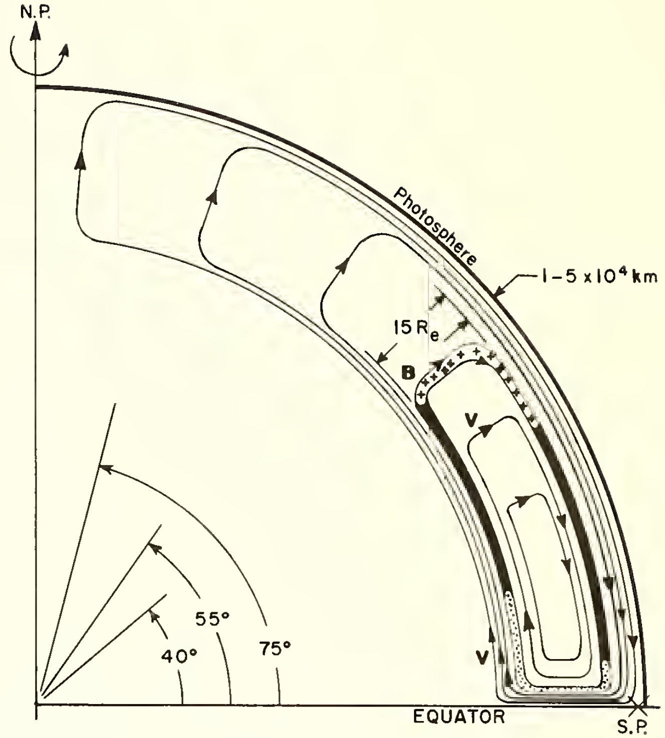
A rather detailed description of the principal magnetic and velocity fields is given in section 2, and a further discussion of meridional circulations is given in section 4. Section 3 contains the formalism relevant to a spherical harmonic analysis of the velocity field over the photospheric surface and shows the observational difficulties involved in ascertaining the poloidal and scaloidal radial functions from which the meridional motions are synthesized; evidently little or no data exists for either the polar or radial velocity components, both of which must be known throughout a radial domain. From an observational point of view, the domain would have to lie above the photosphere and include, say, the chromosphere and base of the corona. But even here we have the difficulty of trying to observe velocities that are just at or beyond our present day precision of measurement. The mean turbulent differential rotation is analyzed into toroidal vectors of odd degree in a straightforward manner, and the analysis here is lacking only in its need for better data in the polar regions. It is expected that eventually a successful surface analysis would supply the boundary conditions for an extrapolation to depths below the photosphere using the hydromagnetic equations. The effects upon the earth's magnetic field of the presence or absence at the upper level of the circulating toroids is discussed in section 5. The region above the upper toroidal field is potentially a zone of convective instability.

2. DESCRIPTION OF THE PRINCIPAL MAGNETIC AND VELOCITY FIELDS

Elaborating on the work of Bullard (1955), figure 1 illustrates roughly the meridional circulations above the photosphere upwards into the coronal layers and below the photosphere to about the bottom of the hydrogen convection zone. In the left-hand figure we have sketched in the salient features of the dynamo without attempting to retain a faithful scale model. Perhaps we have been encouraged into some degree of laxity in this respect because certain of the principal dimensions



Meridional circulation of the sun's toroidal magnetic field.



Streamlines of sun's meridional circulation resemble the poloidal quadrupole velocity mode $\mathbf{P}^{0}_{2c}$, but with augmented speed near the equator.

Figure 1 The sun's principal magnetic and meridional velocity fields.

remain unknown to us. The right-hand figure reproduces more correctly to scale the meridional circulation below the photosphere and therefore justifies our previous imprecision. The figures account for Spörer's law and Hale's law of polarity.

The two zonal toroidal magnetic fields in each hemisphere are indicated by the symbol for magnetic induction, $\underset{\sim}{B}$. Individual lines of force are, in the mean, coaxial circles about the rotation axis (designated by N.P.), lines directed into (or out of) the plane of the page being indicated by plus symbols (or dots). The magnetic toroids are circulated towards the equator at the upper level and towards the poles at the lower level. The toroids descend to greater depths within a few degrees of the equatorial plane and ascend at about $\pm 40°$. We shall refer to the beginning of a sunspot semi-cycle as the time when the sunspots first make their appearance between latitudes 35-40°. At this time the circulating toroid has just ascended into the upper level and begun its equatorial migration, where it is then exposed to convective bombardment by the partially ionized ambient fluid. When convection is sufficiently vigorous, tubes of flux are contorted away from the main toroid and transported upwards through the photosphere to form bipolar spot groups. The spot zone gradually broadens as the toroid at the upper level is circulated towards the equator. The sunspot maximum years occur when the magnetic toroid is fully exposed at the upper level, and sunspot minimum years occur when the upper level is sensibly devoid of the magnetic field and therefore corresponds to a time interval of 3-4 years that overlaps into the beginning of the next sunspot semi-cycle. The average time of rise from sunspot minimum to maximum is about 4.51 years, whereas the decay time from maximum to minimum is about 6.55 years, or about 2 years greater. The existence of the peak or maximum of the sunspot numbers would be accounted for in figure 1 by drawing a slight bulge in the upper toroidal path so that it lies somewhat closer to the photosphere over the higher latitudes, possibly as a result of an augmented radial acceleration during the ascension of the toroid due to magnetic buoyancy.

The half-arrows shown on the periphery of each magnetic toroid in the left-hand side of figure 1 denote the direction of the principal poloidal magnetic fields. These are engendered from the principal toroidal magnetic fields by Coriolis action, the same, we believe, as in planetary dynamos (Parker, 1955), and the transformation, toroidal field into poloidal, is referred to as the <u>regenerative feedback process</u>. Observe that in the Northern Hemisphere the poloidal magnetic field is directed in a way opposite to the electric current system that sustains the toroidal magnetic field, whereas in the Southern Hemisphere they possess the same direction sense. The thickness of the toroidal magnetic sheet is unknown to us as is its depth below the photo-

sphere. The depth shown in the right-hand figure is 41,000 km, about equal to the diameters of the largest sunspots, but the depth may be considerably smaller, say 10,000 km, and there is evidence of a variable height reached by the ascending toroid, from semi-cycle to semi-cycle, according to the variable spot activity and initial latitudes of the occurrence of spots. The widths of the toroids are about 20°, corresponding to the maximum widths of the spot zones during their migration towards the equator. In the right-hand figure the toroids are shown, to scale, circulated to a greater depth by 15 earth radii (15 $R_e \doteq 10^5$ km). This unusually large depth must be postulated to account for the spot zone width and the overlap between semi-cycles, both of which are quite accurately known. Clearly, the fluid velocity is greatest near the equator,and the poloidal quadrupole fluid mode, $\underset{\sim}{P}{}_{2c}^{o}$, that exists below the photosphere is inferred by continuity from the same type of circulation that is known to transport coronal filaments from the regions of the spot zones to the <u>polar crown of filaments</u>. The latter is designated here by the latitude of 75°, but this value fluctuates and the filaments that are formed at the end of a semi-cycle near the equator rarely drift past 45-55° latitudes. The ultimate fate of those filaments that endure to reach the polar crown is, according to figure 1, obscurity by submergence below the photosphere, or complete dissolution.

Polar faculae that are found above latitude 55° are not transported there by secular fluid motions,as is evidenced by their short lifetimes; however, they may possibly be produced from remanent magnetic fields that are transported to the polar regions by a process known as supergranulation, which is a random process in the local velocity field involving convective eddies of larger dimensions; magnetic fields formed in the spot zones presumably migrate upstream (according to our location of the stagnation point) by random motions over the photosphere, to reach the polar regions with a time lapse of a few years (Leighton, 1964). Sheeley (1964) has counted the numbers of north and south polar faculae for the period 1935-1963. He finds a minimum value corresponding to sunspot maximum years, and conversely, so that their numbers are approximately 180° out of phase with the time variation of the sunspot numbers for the whole disk. This work, while corroborating the results of Waldmeir (1955; 1962), and Saitoand Tanaka (1960), extends the observations to a much earlier period. Waldmeir (1955) observed the mean diameter of polar faculae to be 2,300 km and their lifetimes range from a few minutes to several days.

The extent to which the remnant migrating fields contribute to the polar magnetic field is unknown to this writer. The strong correlation between the two regarding polarities and the fact that the polar field reverses at sunspot maximum

when the polar faculae count has its minimum has lead some writers to attribute the polar field solely to the migrating remnant fields themselves, whereas we propose that the most significant part of the polar field ascends to the photosphere from lower depths by the secular meridional circulation.

The polar magnetic fields of about 1 gauss strength are restricted to latitudes above 55°, as indicated in figure 1, and exhibit no persistent obliquity with the rotational axis (Babcock and Babcock, 1955). This latitude restriction is difficult to explain on the hypothesis that the migrating remnant fields are the cause of the polar field. In 1953 both north and south polarities were opposite to that of the earth, and Babcock (1963) established that the reversal of this polar field occurs periodically at sunspot maximum, although the reversal in the two hemispheres is not exactly simultaneous. To account for these phenomena we propose that diffused portions of the principal magnetic fields (especially the poloidal field) are transported by the meridional circulations at the greater depths, to higher latitudes where they emerge near the photospheric level. Further turbulent diffusion and mass emission are then believed to transport the field to chromospheric and coronal altitudes where it is presumably supported by the solar wind and the ambient fluid motions. Point "f" in figure 1, which may assume any heliographic coordinates, shows a closed line fine structure embedded in the more dense regions below the photosphere. The fine structure observed in the polar field may similarly be attributed to turbulence. We recall from Gauss' theorem, however, when assigning the location of any flux tube, that the net flux through the photosphere must vanish.

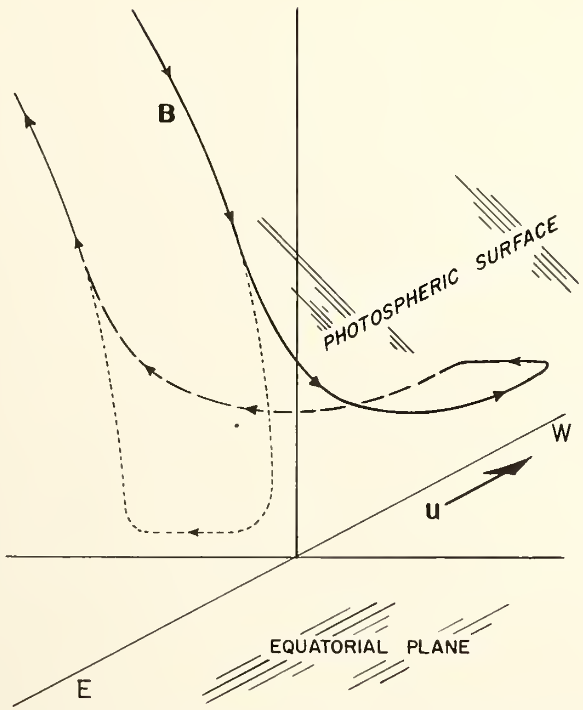
It should also be pointed out that synchronism between the polar field and sunspot cycle should be automatic regardless of the relevant fluid velocities, as long as the magnetic field is periodically injected by diffusion into the meridional circulation.

The true position of the stagnation point of the meridional flow is unknown to us. In both sketches in figure 1 we have tentatively assumed that it lies below the photosphere in order to account, through the Coriolis forces associated with the meridional motions, for the smaller angular velocity inferred to exist at the upper level of the toroidal magnetic field than at the photospheric level. Newton and Nunn (1951) established that large recurrent sunspots rotate more slowly than short lived spots. To explain this observation, we assume that spots are strongly coupled to the lower depths by their own magnetic fields to that a recurrent spot should more more slowly after the ambient fluid has removed the slack from its flux tubes, whereas there would not be enough time for this to occur in short lived spots.

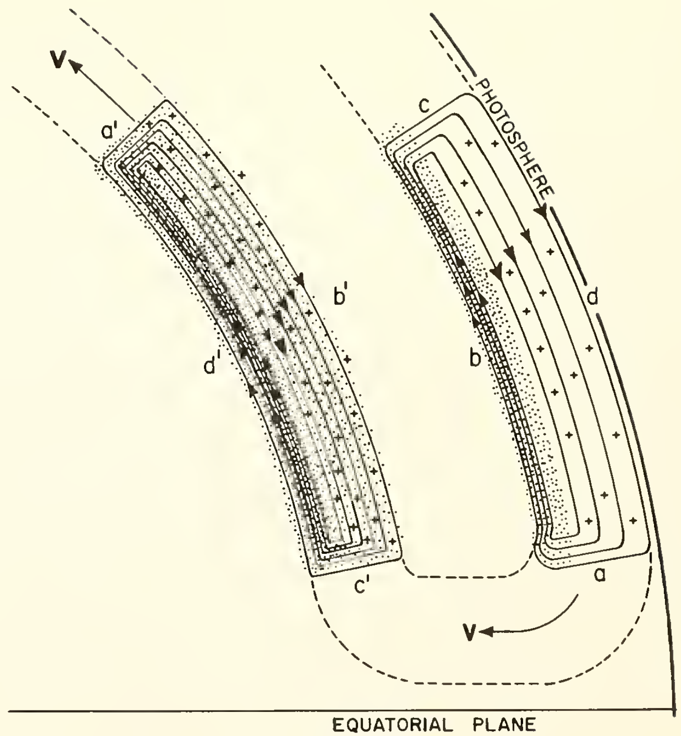
If we place the stagnation point too far below the photosphere it becomes difficult to transport the diffused field to the base of the photosphere to form the polar field.

Previously, the author (1969) described the turbulent differential rotation as a monotone increase in angular velocity as one approaches the rotation axis along a line normal thereto. This follows from the partial conservation of angular momentum, reduced drastically by friction, of the turbulent eddies. In the sun, the photosphere acting as boundary, alters somewhat the azimuthal velocity distribution and the observed equatorial symmetry allows us to analyze the motion into toroidal modes of odd degree, $\underset{\sim}{T}_{1c}^{o}$, $\underset{\sim}{T}_{3c}^{o}$, $\cdots$ But we have previously seen (McDonald, 1969) that these modes engender symmetric meridional motions described by the poloidal fluid modes of even degree, $\underset{\sim}{P}_{2c}^{o}$, $\underset{\sim}{P}_{4c}^{o}$, $\cdots$, and these motions, in turn, further alter the original azimuthal velocity distribution. The problem is further complicated by the chromospheric-coronal circulations. The proper motions of the ambient fluid (as indicated by the motions of filaments) toward the poles augment the angular velocity through Coriolis forces, and these motions, in turn, influence the photospheric layers through viscous diffusion. From the complexity of these motions one should hardly be surprised to find from detailed computation that the stagnation point even lies in the chromosphere. Such a position would seem to favor the larger angular rate observed near the equator, but would make it difficult to account for the smaller angular velocity of recurrent sunspots.

In figure 2, we have sketched the basic solar amplification mechanism. The left-hand figure shows a poloidal magnetic field line being "stretched" in the azimuthal direction by the differential rotation, thereby producing an amplified component of toroidal magnetic field. The photospheric surface is shown simply as a vertical plane, although, as the line of magnetic induction suggests, it should be drawn concave toward the left. The amplification process may be understood from the right-hand figure. The principal poloidal field intensity at the upper level is depicted by the solid closed field lines being greater in the deeper layers where the lines are compacted by compression and where the electrical conductivity and mass density are greater. Suppose for simplicity that the intensity at points "b" and "d" are 30 and 3 gauss, respectively. An amplification by a factor of 100 then produces a toroidal field of 3,000 gauss directed toward the reader at point "b", and one of 300 gauss directed into the plane of the figure at point "d". The latter field is relatively quite negligible and is expected to be cancelled by diffusion of the larger field during the inversion process (a→a', b→b', c→c', and d→d'), in which the configuration is circulated to the deeper levels. During the phase of downward circulation, there is no amplification of the field that is adjacent to the equatorial plane; the field lines are simply transported downstream. After inversion and re-compression the field



Amplification engendered by the θ-dependent turbulent differential rotation velocity **u**. Poloidal field lines are "stretched" in the azimuthal direction.



Amplifying mechanism. A rough sketch of the amplification–transformation of the poloidal magnetic field into the toroidal field. The configuration approaching the equator is circulated to greater depths, and after inversion and compression, it is re-amplified as a similar configuration.

Figure 2 The amplification mechanism of the solar dynamo is based on the turbulent differential rotation and requires a compressible medium.

is re-amplified in the same manner as it was at the upper level, probably, however, to a lesser extent due to its pre-compressed state. Indeed, we recognize that the mechanism would not function in an incompressible fluid of uniform electrical conductivity since there would be no net amplified field.

3. SPHERICAL HARMONIC ANALYSIS OF THE VELOCITY FIELD

Let us write the apparent fluid velocity that is representative of the sun's motions in terms of poloidal, toroidal, and scaloidal vectors, *

$$\underset{\sim}{v} = \sum_{\alpha} (\underset{\sim}{P}_{\alpha} + \underset{\sim}{T}_{\alpha} + \underset{\sim}{L}_{\alpha}) \tag{1}$$

The Greek subscript α is used to designate different sets of identifying indices in the surface harmonics and to indicate whether they contain cos $m\phi$ or sin $m\phi$. The poloidal vectors are defined by the components of $\underset{\sim}{P}_{\alpha} \equiv \nabla \times \nabla \times [\underset{\sim}{\hat{r}} P_{\alpha} (r,t) S_{\alpha}(\theta,\phi)]$, where $S(\theta,\phi), \equiv S_{ns}^{m}$ or S_{nc}^{m}, is the usual spherical surface harmonic, the unnormalized associated Legendre function $P_n^m(\cos\theta)$ multiplied by sin $m\phi$ or cos $m\phi$, respectively. The spherical polar coordinates are denoted by (r,θ,ϕ), in the radial, polar, and azimuthal directions, and the overhead caret symbol denotes a unit vector, in the above case in the r-direction. The toroidal vector is defined as $\underset{\sim}{T}_{\alpha} \equiv \nabla \times [\underset{\sim}{\hat{r}} T_{\alpha}(r,t) S_{\alpha}(\theta,\phi)]$, and the irrotational scaloidal vector is defined by $\underset{\sim}{L}_{\alpha} \equiv \nabla [L_{\alpha}(r,t) S_{\alpha}(\theta,\phi)]$. Scalars P_{α}, T_{α}, and $L_{\alpha}(r,t)$ are arbitrary radial functions.

From (1) we write the instantaneous azimuthal fluid velocity in terms of its poloidal, toroidal, and scaloidal components,

$$r \sin\theta \cdot \dot{\phi}(r,\theta,\phi,t) = \sum_{\alpha} \left[\frac{-T_{\alpha}(r,t)}{r} \frac{\partial S_{\alpha}}{\partial \theta} + \frac{1}{r \sin\theta}\left(\frac{\partial P_{\alpha}}{\partial r} + L_{\alpha}\right)\frac{\partial S_{\alpha}}{\partial \phi}\right] \tag{1a}$$

---

* It should be noted that the divergenceless part of the expansion (1) is deficient in that it cannot be written as the curl of an arbitrary vector. Of course, one may always append a generally small arbitrary vector to (1) in order to satisfy this deficiency, but, in general, the orthogonality properties are all lost unless a judicious choice is made. For example, it is sufficient to append to (1), under the summation sign, the radial vectors $\underset{\sim}{R}_{\alpha} = \underset{\sim}{\hat{r}} R_{\alpha}(r,t) S_{\alpha}(\theta,\phi)$, $n_{\alpha} \geqslant 1$, with arbitrary radial function $R_{\alpha}(r,t)$. This set is automatically orthogonal to the toroidal set of vectors since the latter is deficient in the radial component. It then follows immediately that (2) and (2a) derived below from (1) are also the forms required for this particular complete representation. The properties of these radial vectors and other types of representations will be treated in a separate investigation.

We assume that both axial and equatorial symmetry holds in the time mean. One thus concludes that the mean of the angular (axial) fluid velocity at the photospheric surface, namely, $\dot{\phi}(\theta) \equiv \langle\dot{\phi}(R_p,\phi,\theta,t)\rangle$, must be expressable in the forms

$$\dot{\phi}(\theta)\sin\theta = \dot{\phi}_1 \frac{dP_1}{d\theta} + \dot{\phi}_3 \frac{dP_3}{d\theta} + \dot{\phi}_5 \frac{dP_5}{d\theta} + \cdots \tag{2}$$

$$= a_1 \sin\theta + a_3 \sin 3\theta + a_5 \sin 5\theta + \cdots$$

$$= b_1 \sin\theta + b_3 \sin^3\theta + b_5 \sin^5\theta + \cdots$$

Hence, if we denote by θ_c the heliographic latitude, then the last expression can be written in the form commonly adopted by researchers:

$$\dot{\phi}(\theta) = c_0 + c_2 \sin^2\theta_c + c_4 \sin^4\theta_c + \cdots \tag{2a}$$

The sets $\{\dot{\phi}_i\}$, $\{a_i\}$, $\{b_i\}$, $\{c_i\}$, are constants over any spherical surface to be determined from the observed meaned data. For an actual SH analysis of the sun, however, all the coefficients must be converted to the set $\{\dot{\phi}_\alpha\} = \{-T_\alpha(r)/r^2\}$.

The solar rotation has been estimated by numerous observers from measurements made on the movements of sunspots across the disk or from spectroscopic determinations of the Doppler shift, when viewing the solar limbs, particularly. One serious objection to former analyses is that other forms than (2) or (2a) have been used so that some error has been introduced in choosing arbitrary functions that are inadequate representations. Other writers have used form (2a) but have truncated the series after the quadratic term.

Scheiner in 1630 was the first to observe that spots near the equator had shorter revolution periods than those at higher latitudes. This latitude dependency was first established by Carrington in 1863 from observations made on sunspots. He obtained an equatorial sidereal period of 24.96 days corresponding to a daily angular velocity of 14.42°. At latitude 35°, he found the angular velocity was reduced by about 1°/day. Passing now over the vast amount of literature that has since accumulated--see the recaptiulation by De Lury (1939)--we come to a rather recent result by Newton and Nunn (1951) who summarized the results obtained at Greenwich from recurrent groups of spots, mostly unipolar, covering six semi-cycles from 1878 to 1944, observing principally the time of central meridian passage. Their sidereal daily angular velocity law

was given in the form

$$\dot{\phi} = 14.38^{\circ} - 2.77^{\circ} \sin^2\theta_c = 11.61^{\circ} + 2.77^{\circ} \sin^2\theta; \; |\theta_c| \leqslant 35^{\circ}, \tag{3}$$

corresponding to an equatorial linear velocity of about 2 km sec^{-1}. This expression was obtained by fitting angular rates meaned over 5° latitude belts up to 35° in both hemispheres. Using data from non-recurrent sunspots for the period 1934-1944, values of the daily sidereal motion were derived also from 5-day means of longitude. Mean values for 5° belts of latitude were again derived up to 35° to obtain

$$\dot{\phi}_o = 14.41^{\circ} - 3.36^{\circ}\sin^2\theta_c; \; |\theta_c| \leqslant 35^{\circ}, \tag{3a}$$

thus demonstrating the more rapid motion of non-recurrent spots. Further, they found no measurable variation in the sun's rotation period, neither with alternate semi-cycles nor fractions thereof, inferring that the periodic presence or absence of the underlying toroidal magnetic field does not measurably alter the angular velocity at the photospheric levels. Newton and Nunn (1951) also point out that measures of solar rotation from sunspot observations by different observers are in close agreement, the total deviation in the range of equatorial velocities being only 0.22°/day, corresponding to a deviation in rotation period of about 0.4 days.

Goldberg (1953), however, in summarizing the results of spectroscopic observations, points to a possible contradiction of the above conclusion; there seem to be systematic changes in the velocity as measured by the same observer over a period of years, which have sometimes been attributed to real variations in the period of solar rotation, perhaps correlated with the sunspot cycle. Hence, the problem does not seem to be entirely settled. Even the recent spectroscopic observations by Livingston (1969) indicate questionable dissymmetries; while verifying the discovery made by Adams, in 1911, that the sun rotates about 0.5°/day faster in H_{α} than in the metallic lines, his observations showed that for the first half of 1967 the Southern Hemisphere lagged the Northern in H_{α} but not in Fe 5250 or Fe 5233. Spectroscopic determinations, however, have been much more discordant than those involving the direct observations of semi-permanent features over the solar surface and significant variations of period of rotation at any given latitude are probably nonexistent. The accuracy of measuring tangential velocities by spectroscopic methods is about 0.01 km sec^{-1}.

An immediate objection to the method of observing semi-permanent features on the solar disk, such as spot groups, is that their movement is not truly representative of the motion of the ambient fluid. Spots often exchange material with their surroundings

and hence undergo a reactive drift velocity. Also, the centroid of a spot group often changes during the development or decay of the group, or a selected spot within a group may move relative to the remaining spots, or additional small spots may appear, etc. Sometimes two spots are seen to interpenetrate each other (being at different radial levels), which vividly shows their shortcoming as a true indicator of azimuthal motion. Of greatest importance, however, is their magnetic attachment to the lower layers, such that after several rotations, the slack being removed from their tubes of force, their motion is sensibly that of the layers from which they were derived.

De Lury (1939) observed that the average velocity of (recurrent or large) sunspots is about 1% less than the average of faculae and flocculi and that the difference changes little with latitude. Ward (1966) pointed out, however, that this discrepancy vanishes when one recognized that the larger spots have rotation rates 1% smaller than those of small area. Ward (1965;1966) presents rotational rates that were computed from the daily longitudinal proper motions of sunspot groups, using data from the Greenwich Photoheliographic Results. He observed that small sunspot groups whose longitudinal dimension is many times the latitudinal dimension, move up to 2% more rapidly in azimuthal motion than those of large, roughly circular, cross-section. A comparison between the results of Ward (1966) and Newton and Nunn (1951), using their results (3), is made in the below tabulations which show the mean value of the rate of solar rotation at 5° latitude intervals, in units of degrees per day:

LATITUDE INTERVAL						
0-5	5-10	10-15	15-20	20-25	25-30	$\geqslant$30
Newton-Nunn, 1878-1944, long-lived spots						
14.37	14.33	14.25	14.13	13.97	13.79	---
Ward, 1905-1954, all spot groups						
14.52	14.47	14.39	14.29	14.14	13.94	13.71

The difference in rotational rates of different types of spots, discovered by Newton and Nunn, is closely related to a phenomena discovered and explained by Maunder (1907), who found that at any time: (a) there are more spots visible on the apparent west side than on the apparent east side of the solar disk, and (b) more spots form on the apparent west half of the disk and more spots enter the apparen west limb by rotation than disappear on the apparent east limb. These observed east-west differences were explained as a foreshortening effect of spots whose axes are tilted from the vertical. By observing the spot area at successive intervals before and after central meridian passage one can readily compute the angle of tilt. Recent observations, however, by Haurwitz et al. (1970) failed to disclose a mean tilt of spot axes so that the answer to the apparent west excess seems to be unsettled; Gleissberg (1940) found a mean tilt of 0.6° leaning toward the apparent east, and

Minnaert (1946) subdivided the spots by their age and found an average tilt of 0.44° for all spots (average age about 1 day) whereas recurrent spots of age equal to or greater than 3 disk passages showed a mean tilt of 7.6° for the intermediate passage. This is consistent with a solar rotation rate that increases slightly with height above the magnetic toroids, upwards into the corona--contrary to the velocity gradient that would be engendered by a turbulent differential rotation without meridional flow. The coriolis forces associated with the meridional motions must contribute significantly to this behaviour. The large recurrent spots are magnetically coupled to the lower layers so that it seems reasonable that the magnetic tubes would become tilted as the spot is dragged by the ambient fluid. Accordingly, recurrent spots would move slower than short-lived spots that do not endure long enough to have the slack taken out of their (weaker) magnetic tubes. Similarly, at the greater depths to which the toroids are circulated the coriolis forces would lead to an augmented rotation velocity.

A second explanation of the east-west effect,that does not depend on the spot tilt, is introduced by the asymmetry of the spot group itself. The leader or preceeding (p) spot is usually larger and more compact than the following (f) spot and its predominence increases with age of the group. Indeed, the majority of unipolar spots were initially bipolar, the p-spot being the sole survivor of the group. This causes an increased visibility of the entire group when on the apparent western half of the disk.

An alternate expression for the daily angular motion was deduced from extensive measurements by d'Azambuja and d'Azambuja (1948) on coronal filaments,

$$\dot{\phi}(\theta) = 14.42^{\circ} - 1.40^{\circ}\sin^2\theta_c - 1.33\,\sin^4\theta_c + \cdots \qquad (4)$$

$$= 11.69^{\circ} + 4.06\,\sin^2\theta - 1.33\,\sin^4\theta + \cdots.$$

Recalling that filaments generally reach up to several thousand kilometers above the photospheric surface and possess a poleward proper motion, their angular rates should surpass those of spots, because of Coriolis action. They found that the angular equatorial velocity of spots is somewhat smaller than that of filaments and that the space gradient towards the pole is also slightly less for filaments. They conclude that the angular velocity in the axial direction is fairly independent of the radial coordinate throughout the chromosphere and well up into the corona, and that these various solar layers accessible to observation have sensibly equally increasing space gradients with respect to the polar angle.

Observations at even higher altitudes, in the solar electron corona from 1.125 R_p to 2.0 R_p, have been made by Hansen, Hansen, and Loomis (1969); localized coronal features that are 2-3 times brighter than the quiescent corona are observed during several successive rotations.

We observe from (4) that the difference in rotation rates between the equator and the 30th parallels is about 12^0 per rotation, which is equivalent to a differential rotation period of 810 days. This is about 10^{-3} times the differential rotation periods expected in the earth's core, and thus accounts for the much larger magnetic fields found in the sun.

Waldemeir (1955) has obtained the solar rotation rate by observing the motions of longer-lived polar faculae across the disk. Polar faculae were observed from the morning of the first day until the evening of the second day, and some even over 3 days. The daily (sidereal) rotation rates for 8 distinct polar facular points are tabulated below, together with the predictions of (4):

No.	Helio.Lat.	$\dot{\phi}$ Observed	$\dot{\phi}$ from (4)	No.	Helio.Lat.	$\dot{\phi}$ Observed	$\dot{\phi}$ from (4)
1	-67^0	10.9^0/day	12.28^0/day	5	-74^0	9.8^0/day	11.99^0/day
2	-68	10.4	12.23	6	-74	10.5	11.99
3	-70	10.0	12.15	7	-75	9.7	11.96
4	-70	10.4	12.15	8	-80	10.2	11.81

Comparison indicates that the predicted values are too large over the polar caps. On the other hand, large discrepancies exist in the observed values; facular points Nos. 6 and 8 are larger than the rest of the data would indicate and a least squares fit would clearly be attended by a large standard deviation. It is suggested that an improved fit be constructed by adding higher degree terms to (4) and evaluating the coefficients to account for improved observations over the polar caps.

Let us next compute the radial functions on the assumption that the necessary velocity distributions are known over the outer solar domain. In estimating the radial and north components of velocity, some information can be obtained from the migration of the sunspot zones and the spot reversal period. The θ-distribution of azimuthal velocity determines the toroidal component modes. Thus from the established orthogonality of these basic vector sets (See, for example, McDonald and Stearns, 1969), we multiply (1) by $\underset{\sim}{T}_{\alpha'u}$ to form their scalar dot product, where $\underset{\sim}{T}_{\alpha'u}$ is a toroidal vector of unit radial function for all r. Next integrate over unit sphere to obtain the required radial functions of toroidal type,

$$T_\alpha(r,t) = \frac{r}{N_\alpha} \oiint \left[-v_\phi \frac{\partial S_\alpha}{\partial \theta} + \frac{v_\theta}{\sin\theta} \cdot \frac{\partial S_\alpha}{\partial \phi}\right] \sin\theta \, d\theta \, d\phi, \tag{5}$$

where the normalization factor is, for $n \geq 0$,

$$N_\alpha = N_n^m \equiv (1+\delta_o^m) \frac{2\pi n(n+1)}{2n+1} \cdot \frac{(n+m)!}{(n-m)!}, \quad m \leq n, \text{ where } \delta_o^m = 0 \text{ when } m>0; \; \delta_o^o = 1.$$

Write $r = R_p$ when evaluated at the photospheric surface. Forming the ensemble average we recognize that $\langle v_\theta \rangle$ is independent of ϕ so that the last term vanishes and we obtain, with $\langle v_\phi \rangle = r \sin\theta \, \dot{\phi}(\theta)$ and $n_\alpha = 1, 3, 5, \cdots$,

$$\langle T_\alpha(r,t) \rangle \equiv T_\alpha(r) = -\frac{r}{N_\alpha} \oiint \langle v_\phi(r,\theta,\phi,t) \rangle \frac{\partial S_\alpha}{\partial \theta} \sin\theta \, d\theta \, d\phi = \qquad (6)$$

$$= -\frac{2\pi r^2}{N_\alpha} \int_0^\pi \dot{\phi}(\theta) \frac{\partial S_\alpha}{\partial \theta} \sin^2\theta \, d\theta .$$

Symmetry considerations require that $T_\alpha(r) = 0$ when n_α is even. Substituting $\dot{\phi}(\theta)$ as given by (4), and making use of the auxiliary tabulations that express the value of the definite integral $I_{2n+1} \equiv \int_0^\pi \sin^{2n+1}\theta \, d\theta$, namely,

n	I_{2n+1}	n	I_{2n+1}
1	1.333,333,3	4	0.812,698,4
2	1.066,666,7	5	0.738,816,7
3	0.914,285,7	6	0.681,984,7

we obtain for the toroidal radial functions at the photospheric surface (with R_p = 7.0×10^{10}cm)

$T_{1c}^o(R_p) = 1.38833 \times 10^{16}$ cgs (dimensions L^2T^{-1})

$T_{3c}^o(R_p) = -0.0301789 \times 10^{16}$ cgs

$T_{5c}^o(R_p) = -0.00334404 \times 10^{16}$ cgs

$T_{7c}^o(R_p) = +0.00008 \times 10^{16}$ cgs .

The first coefficient, T_{1c}^o, corresponds to a rigid body rotation of 14.026^o/day. Had we used the analysis of Newton and Nunn (3) we would have obtained a rigid body rotation of amount 13.826^o/day instead. Recall, however, that their result is appropriate only to the lower latitude belts. The reader may verify that these angular rates give an equivalent angular momentum in axial direction over the photospheric surface of amount $N_\alpha T_\alpha$, for $n_\alpha = 1$. From the above four coefficients we may expect to find a rapid rate of convergence.

Next, consider the poloidal and scaloidal components. Form the scalar dot product of $\underset{\sim}{P}_{\alpha u}$ with each term in (1) and integrate over unit sphere. We find, after forming

the ensemble average and further writing $\langle v_\theta \rangle = v_\theta(r,\theta)$, $\langle v_r \rangle = v_r(r,\theta)$,

$$\frac{n(n+1)}{r^2} P_\alpha(r) + \frac{dL_\alpha(r)}{dr} = \frac{2\pi n(n+1)}{N_\alpha} \int_0^\pi v_r(r,\theta) \sin\theta S_\alpha d\theta; \; m_\alpha = 0, \tag{7}$$

$$\frac{n(n+1)}{r^2} P_\alpha(r) + \frac{dL_\alpha(r)}{dr} = 0; \; m_\alpha > 0.$$

Carrying out a similar operation on (1) with the scaloidal function $\underset{\sim}{L}_{\alpha u}$, of unit radial function, and requiring zonal symmetry, we find

$$\frac{dP_\alpha(r)}{dr} + L_\alpha(r) = \frac{2\pi}{N_\alpha} \int_0^\pi r v_\theta(r,\theta) \frac{dS_\alpha}{d\theta} \sin\theta d\theta; \; m_\alpha = 0, \tag{8}$$

$$\frac{dP_\alpha(r)}{dr} + L_\alpha(r) = 0; \; m_\alpha > 0.$$

For $m_\alpha > 0$, (7) insures against any contribution, with a ϕ-dependency, to the radial velocity, and (8) insures against similar $\underset{\sim}{\hat{\theta}}$- and $\underset{\sim}{\hat{\phi}}$-component velocity contributions. Evidently it is sufficient to choose $L_\alpha(r) \equiv P_\alpha(r) \equiv 0$, although any simultaneous solutions of the abridged equations of (9) and (15) are acceptable. Solving the first two equations in (7) and (8) for the dependent variable $P_\alpha(r)$, we obtain the differential equation

$$-P_\alpha^\dagger \equiv \frac{d^2P_\alpha}{dr^2} - \frac{n(n+1)}{r^2} P_\alpha = f_\alpha(r), \tag{9}$$

where

$$f_\alpha(r) \equiv \frac{2\pi}{N_\alpha} \int_0^\pi d\theta \sin\theta \left[\frac{\partial}{\partial r} (r v_\theta) \cdot \frac{\partial S_\alpha}{\partial \theta} - n(n+1) v_r S_\alpha\right]. \tag{9a}$$

Extensive use of the symbol $P_\alpha^\dagger$ has been made by the author (1969) in transforming the Navier-Stokes equations. It follows that $v_r(r,\theta)$, $v_\theta(r,\theta)$, and $r\partial v_\theta/\partial r$ must be known throughout a spherical-shell domain in order to compute P_α. Direct observation presently restricts us to the domain above the photosphere. The general solution of (9) is given in terms of the iterated doubly indefinite integral,

$$P_\alpha(r) = \Phi_\alpha \int^r \frac{dr}{\Phi_\alpha^2} \int^r f_\alpha \Phi_\alpha \, dr, \tag{10}$$

where $\Phi_\alpha(r)$ is the general solution of the abridged equation of (9),

$$\Phi_\alpha = r^{\frac{1}{2}} \left[A_\alpha r^{\frac{1}{2}\sqrt{1+4n(n+1)}} + B_\alpha r^{-\frac{1}{2}\sqrt{1+4n(n+1)}}\right], \tag{11}$$

with A_α, B_α entirely arbitrary parameters. One may readily demonstrate the validity of (10) by successive integration by parts while using the abridged equation of (9); we see that we recover $P_\alpha(r)$ regardless of the parameter values assigned to A_α and B_α. Thus for simplicity we may choose $A_\alpha = 0$ so that B_α then cancels in (10); conversely, we may take $B_\alpha = 0$, with A_α cancelling.

At the origin of coordinates we require the fluid velocity $\underset{\sim}{v}$ and its rectangular components to be uniquely defined and continuous, since we do not expect it to be a vector singularity. But the origin is a singular point of the spherical coordinate system so that $v_r(r,\theta)$ and $v_\theta(r,\theta)$ are not uniquely defined at $r = 0$, but must be continuous together with their radial derivatives of lower order as we approach the origin along any polynomial curve, and in particular, along any radial line. Therefore, we require that $r^{-2}\, P_\alpha(r)$, $r^{-1}\, \partial P_\alpha/\partial r$, $\partial L_\alpha/\partial r$, $r^{-1}\, L_\alpha$ be continuous at $r = 0$. From (9) it then follows that $f_\alpha(r)$ is continuous at all points $0 \leqslant r \leqslant \delta$ and possesses a derivative at all interior points. Hence, we may write $f_\alpha(r) = f_\alpha(0) + r f_\alpha'(\xi)$, $0 < \xi < r$, and evaluate (10) directly by using the first mean-value theorem for integrals (Confer Jeffreys and Jeffreys, 1950). From straightforward computations we find the poloidal radial velocity component at $r = 0$ to be

$$\lim_{r\to 0} \left[\frac{n_\alpha(n_\alpha+1)}{r^2} P_\alpha(r)\right] = \frac{n_\alpha(n_\alpha+1) f_\alpha(0)}{2 - n_\alpha(n_\alpha+1)}; \; n_\alpha = 2,3,\cdots; \; m_\alpha = 0 \tag{12}$$

$$= 0, \text{ only when } n_\alpha = 1; \; m_\alpha = 0.$$

The computations are easier to carry out by setting $A_\alpha = 0$. When B_α is set equal to zero and we employ the first factor in (11), we encounter an unbounded result only when $n_\alpha = 1$, showing that we must have $f_\alpha(0) = 0$ in this case. This result means that the zonal dipole term, $\underset{\sim}{P}{}_{1c}^{0}$, must vanish at $r = 0$. This stringent restriction results from a lack of generality of the form (1) chosen for the representation of the velocity field. Nevertheless, because of the assumed equatorial symmetry in the time mean we must have $f_\alpha(r) \equiv 0$ in (9a), for all odd integers, $n_\alpha = 1,3,5,7\cdots$. Therefore, the above conclusions are harmonious with our postulated model and we may quite superficially write $n_\alpha = 2,4,6,\cdots$, in (12). Using (9) we also find that for $m_\alpha = 0$,

$$\left(\frac{d^2 P_\alpha}{dr^2}\right)_{r=0} = \frac{2 f_\alpha(0)}{2 - n_\alpha(n_\alpha+1)}; \; n_\alpha = 2,4,6,\cdots \tag{13}$$

Other quantities may be similarly evaluated at $r = 0$.

To compute the scaloidal components we may solve for L_α in (8) and compute dP_α/dr by differentiating (10):

$$L_\alpha(r) = -(1/\Phi_\alpha) \int^r f_\alpha \Phi_\alpha \, dr - \frac{d\Phi_\alpha}{dr} \int^r \frac{dr}{\Phi_\alpha^2} \int^r f_\alpha \Phi_\alpha \, dr \tag{14}$$

$$+ \frac{2\pi}{N_\alpha} \int_o^\pi r v_\theta \frac{dS_\alpha}{d\theta} \sin\theta d\theta ; \; n_\alpha \geqslant 1, \; m_\alpha = 0.$$

Also, from (7) and (8), one may readily verify that $L_\alpha(r)$ must satisfy the differential equation,

$$\frac{d}{dr} (r^2 \frac{dL_\alpha}{dr}) - n(n+1) L_\alpha =$$

$$= \frac{2\pi n(n+1)}{N_\alpha} \int_o^\pi d\theta \sin\theta \left[\frac{d}{dr} (r^2 v_r) S_\alpha - r v_\theta \frac{dS_\alpha}{d\theta}\right], \tag{15}$$

again showing that the radial dependency of v_r and v_θ must be known to effect a solution from observed data. As previously mentioned, the magnitudes of these mean conponents are below the present day precision of measurement. The presence of short-lived convective eddies with velocities 2-3 orders of magnitude larger than these expected mean motions augments the difficulty of making direct observations. Evidently an approximate approach to this problem is to infer the velocity from our knowledge of the circulating magnetic toroids, but this is a poor substitute for the solution of the boundary-value problem at hand. Further knowledge of the large-scale secular motions may be expected from computer-type analyses of the hydromagnetic equation involving specialized configurations (See, e.g., Davies-Jones and Gilman, 1969).

4. MERIDIONAL CIRCULATIONS

The meridional circulating systems at or below the photospheric level comprise poloidal and scaloidal fluid modes of even degree with motion towards the equator, where a general subsidence occurs, and upwelling near both rotation poles. The general circulation resembles the poloidal quadrupole mode, $\underset{\sim}{P}{}_{2c}^{0}$, but with augmented modulus near the equator. Subsidiary modes of this type are postulated in the corona to account for the poleward proper motions of filaments at all latitudes up to the polar crown of filaments.

The fluid velocity in the meridional circulation is estimated by noting that the sunspot cycle has a period of about 22.5 years, while the circuit extends to latitudes of about $\Delta\theta = 40^{0}$ and differential depths of about 12-15 earth radii. The mean velocity of the equatorial migration of the toroidal magnetic fields is accordingly,

$$\frac{R_p\Delta\theta+12R_e}{\frac{1}{2}\mathrm{x}22.5\ \mathrm{yrs}} = 1.59\ \mathrm{m\ sec}^{-1}. \qquad (16)$$

This value is reasonably consonant with the observed larger mean poleward velocity of filaments ($\pm$ 1^{0}/rotation, or 5 m $\sec^{-1}$) whose heights are found to range up to 25,000 to 100,000 km above the photosphere; at such altitudes the equation of continuity would demand larger velocities in the coronal circulation because of the smaller mass density relative to the photospheric value.

The results of M. and L. d'Azambuja (1948) on the poleward drift of filaments at various latitudes shows that there is no reversal of sign at high latitudes, but that the drift rate diminishes monotonically with latitude until one reaches the diffuse upper boundary, the so-called <u>polar crown</u> of filaments, previously mentioned, whose latitudes near $\pm$ 70^{0} fluctuate with the spot cycle. The poleward drift rate varies from about 2^{0}/rotation (or 10 m $\sec^{-1}$) at latitudes $\pm$ 10^{0} to about 1^{0}/rotation (or 5 m $\sec^{-1}$) at latitudes $\pm$ 45^{0}, and 0.8^{0}/rotation (or 4 m $\sec^{-1}$) at latitudes $\pm$ 65^{0}. [The possibility of a variation in these velocity rates has been expressed by the d'Azambujas (1948), who claim that the drift rate is largest during the ascending part of the sunspot semi-cycle.] Magnetic fields inside prominences, or filaments, measure upwards to 200 gauss. Thus, they may not be entirely detached from the photospheric layers so that the velocity of the ambient fluid may be somewhat larger than their observed poleward proper motions. Incidentally, prominences tend to become more

aligned in an east-west direction during successive rotations, thus confirming that they are acted upon by a θ-dependent differential rotation, assumed sensibly identical to the azimuthal velocity gradient at the photospheric level.

Any birth of filaments at latitudes higher than the spot zones is unknown to us and filaments that form outside of spot groups invariably form in facular regions. At sunspot minimum the magnetic toroid of the new phase is radially ascending and making its appearance at about the 35-40^{0} parallels where new prominences are produced. These subsequently drift poleward and after about 3 years reach the 70-80^{0} latitudes, thus enduring some 30-40 solar rotations. As the sunspot zone migrates toward the equator the maximum latitudes to which the prominences subsequently drift, also decreases; near the end of the semi-cycle, prominences are produced at about 5-15^{0} latitudes and after about 3 years drift only to the 45-55^{0} latitudes before complete dissolution or obscuration.

Waldmeir (1939), as summarized by Kiepenheuer (1953), has shown that when the maximum monthly smoothed value of sunspot number during the semi-cycle is larger, then the latitude at which the migration begins is larger. These observations suggest that a fluctuation exists in the heights reached by the ascending magnetic toroids (figure 1) at the beginning of successive semi-cycles, a closer approach to the photosphere resulting in a larger number of sunspots and a higher latitude at which migration of the spot zone begins, as previously mentioned on page 4.

Any evidence of the depth of the outermost toroid below the photosphere seems to be lost to us through the action of the turbulent differential rotation, which appears to predominate over the Coriolis forces acting on each upwelling. The preponderance of near east-west orientations of the p and f spots in a group suggest a shallow depth of the same order of magnitude as the dimensions of the spot, but what is difficult to explain is that almost all spot groups are oval with the long axis slightly orientated away from the east-west direction such that the p-spot is closest to the equator. This orientation angle increases with latitude to a maximum of about 19^{0} and decreases with the age of the spot group, probably as a result of the action of the solar differential rotation (Kiepenheuer, 1953).

The meridional velocities at the photospheric level are too small to be observed by present day methods. Plaskett (1966) carried out spectroscopic measurements of 190 photospheric positions throughout the latitude range 45-75^{0}, north and south. After due consideration of former observations that he had reported on (Plaskett, 1959;1962), one of his several conclusions was that the meridional circulations were directed toward the equator in both hemispheres (there also appeared evidence for a hemispheric

difference of rotation rates.) His observed velocities, however, are at least an order of magnitude larger than those expected--surely not more than 5 m sec^{-1}, the mean poleward drift rate of coronal filaments. Moreover, his probable errors, which are about equal to his observed velocities and which result partly from rather persistent local velocity fields, are sufficiently large that for latitudes above $\pm$ 45^{0} the photospheric velocity could be zero or even directed poleward by as much as 5 m sec^{-1}.

The phase-synchronism of the meridional circulations neighboring the equator and manifested by the ever near simultaneous occurrence of the sunspot maximum and minimum phases in the N. and S. Hemispheres, is maintained sensibly by the electromagnetic ponderomotive forces. With incipient disorder, the circulating toroidal magnetic field that is leading engenders Lorentzian forces, by self-interaction, that tend to force the assemblage across the equatorial plane, thereby delaying its forward movement by increasing the circuital path length. Smaller ponderomotive forces are also engendered by interaction of the electric current systems that sustain the toroidal fields with the associated poloidal fields and the poloidal fields on opposite sides of the equator. These forces, however, being in the aximuthal direction, must engender further Coriolis action to be effective in bringing about the required synchronism. Also, the smaller interaction forces between the electric current system that sustains each toroidal field and the toroidal field on the opposite side of the equator opposes the movement that brings the two toroids into closer proximity, thereby tending to keep the toroids separated.

5. THE SUNSPOT ZONE—A REGION OF CONVECTIVE INSTABILITY

An interesting comparison is made in figure 3 between the geomagnetic fluctuations observed during sunspot minimum and maximum years. The figure shows the plots of the monthly means of the hourly values of the horizontal intensity, H, taken from selected observatory records and representing, generally, five international quiet days of each month or ten selected local quiet days, as adopted by the United States. All fluctuations with periods less than about 1 year are of external origin, since the conducting mantle, acting as a low pass filter, virtually suppresses such rapid fluctuations that may be engendered in the core. Several European stations of close proximity (not all shown) were selected in order to estimate the highest degree of correlation between different stations that one may expect to find under actual conditions. To ascertain whether a perturbation was of global extent, comparison was made with observations at Hermanus, Agincourt, Amberly, and stations operated by the United States (not all shown). The curve for each station was roughly sketched through the data

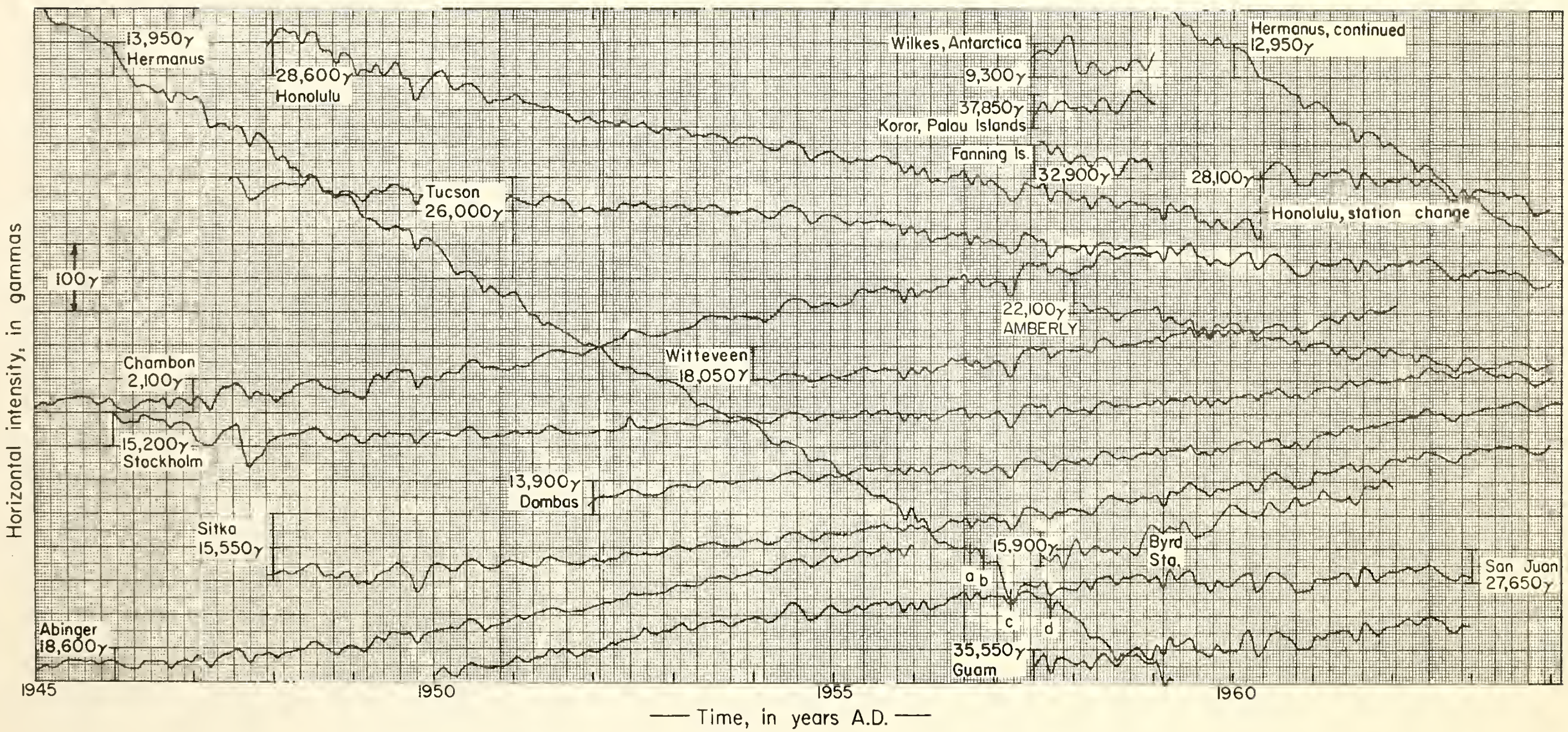


Figure 3 Monthly means of the hourly values of horizontal intensity from selected geomagnetic observatories.

points, plotted at the middle of each month.

During sunspot minimum years we see clear evidence of the annual (periodic) variation, AV. Thus in 1951 through 1955 one sees an augmented intensity of some 10-20 gammas during the summer months at Stockholm, Abinger, and Chambon.

Also of considerable interest are plots of the radial intensity, B_r (not shown). In the N. Hemisphere perturbations in B_r and H vary virtually in time phase whereas they differ by π radians in the S. Hemisphere. Equatorial stations of low geomagnetic latitude show little or no response to large perturbations, whereas high latitude stations show radial fluctuations about equal to the horizontal fluctuations. This is precisely the response that one would expect from an equatorial ring current located at a few earth radii and perturbed by the arrival of solar corpuscles.

The augmented global fluctuations that characterize the sunspot maximum years, 1946-1949 and 1957-1960, ascertained by counting numbers of sunspots, offer an immediate contrast to the smoother sunspot minimum features. The time marks a, b, c, d, that are superimposed on Hermanus typify global depressions in the well-correlated horizontal component. During sunspot maximum years the larger H-fluctuations of 30-40γ (Witteveen in 1957, etc.), occur with periods ranging from about 0.2 to 0.5 years, the onsets of disturbance being virtually simultaneous. A depression of fully two months was first noted by Bower in 1908. Mean global rates of change of 200-300γ/yr are associated with these large fluctuations. The largest perturbation in 1957, designated by time mark "c" passes through its minimum value "continuously" and persists for several months. Examination of continuous-run observatory records reveals that the deeper global H-depressions are due to a near rash of magnetic storms continuing throughout the lifetime of the depression, and the inference is made that they are associated with active regions on the sun that occur at random geographical positions; they cannot be associated principally with recurrences of large spot groups, but with distinct groups. The phenomenon appears to result from disturbances at great depths in the sun, possibly in the hydrogen reactor itself, the time-dispersion of the perturbations at the surface resulting possibly from different turbulent paths.

In contrast to the above phenomena, depressions may occur during sunspot minimum years without being associated with magnetic storms. Thus, the depression centered at September 1954 is about as deep as the depression designated by time mark "d" in September 1957. Comparing the continuous-run observatory records for Tucson, however, we find no storms over 100γ in September 1954, whereas in 1957 we date the following storms together with their approximate maximum depressions: 29 Aug., 150γ; 2 Sept., 200γ; 4 Sept., 350γ; 12 Sept., 400γ; 21 Sept., 150γ; 22 Sept., 250γ; 28 Sept., 250γ; 13 Oct., 100γ. Evidently the presence of the magnetic toroid, stretched out

across the sunspot zone, has a significant effect on the thermal convection by which the toroid is continually bombarded.

We have previously described how the region above the toroidal field is a zone of instability (McDonald, 1969, pp. 52-53). Similarly, here the modulus of the toroidal magnetic field $\underset{\sim}{B}_T$ is large and decreasing, $\partial B_T = \partial r < 0$, as one approaches the photosphere from below, and complete inhibition of convective cells smaller than a critical diameter again is expected. The lapse rate above the upper toroid is similarly increased by the presence of the magnetic field and the convection that does break through does so with augmented vigor. Nevertheless, as the above H-depression in 1954 indicates, corpuscular emission takes place even when the magnetic toroid is not stretched across the spot zone. The extent to which this occurs in active regions--say, invisible spots--that may not have been discerned because of the sensible absence of the magnetic toroid, is unknown to us.

6. ACKNOWLEDGEMENT

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